Solutions to Problem Sheet 1 (Topics 1 - 4)

1. Circuit (a) is a parallel circuit: there are only two nodes and all four components are connected between them.

    Circuit (b) is a series circuit: each node is connected to exactly two components and the same current must flow through each.

2. For - B the voltage and current correspond to the passive sign convention (i.e. the current arrow in the opposite direction to the voltage arrow) and so the power absorbed by B is given by V x I = 20W.

    For device A we need to reverse the direction of the current to conform to the passive sign convention. Therefore the power absorbed by A is V x I = - 20W.

    As must always be true, the total power absorbed by all components is zero.

3. The power absorbed is positive if the voltage and current arrows go in opposite directions and negative if they go in the same direction. So we get: (a) P_V = +4, P_I = - 4, (b) P_V = +4, P_I = - 4, (c) P_V = -4, P_I = +4, (a) P_V = -4, P_I = +4. In all cases, the total power absorbed is P_V + P_I = 0.

4. We can find a path (shown highlighted below) from the bottom to the top of the V_X arrow that passes only through voltage sources and so we just add these up to get the total potential difference: V_X = (-3) + (+2) + (+9) = +8V.
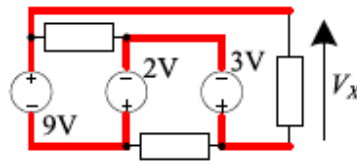
5. If we add up the currents flowing out of the region shown highlighted below, we obtain I_X − 5 − 1 + 2 = 0. Hence I_X = 4 A.
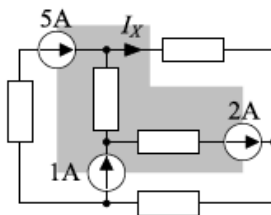
6. The three series resistors are equivalent to a single resistor with a value of 1 + 5 + 2 = 8 kΩ.

7. The three series resistors are equivalent to a single resistor with a value of

    $$\frac{1}{1/1+1/5+1/2} = \frac{1}{1.7}$$ = 0.588 kΩ.

8. We can first combine the parallel 2 kΩ and 3 kΩ resistors to give   (2x3)/(2+3) = 1.2kΩ. This is then in series with the 4 kΩ resistor which makes 5.2 kΩ in all. Now we just have three resistors in parallel to give a total of 1/(1/1 + 1/5 + 1/5.2) = 1/1.39 = 0.718 kΩ.

9. The resistance is 1/8 x 10^{-6} = 125 kΩ.

10. [Method 1]: The resistors are in series and so form a potential divider. The total series resistance is 7 kΩ, so the voltages across the three resistors are 14 x 1/7 = 2V, 14 x 2/7 = 4V and 14 x 4/7 = 8V. The power dissipated in a resistor is V^2/R , so for the three resistors, this gives $2^2/1$ = 4mW, $4^2/2$ = 8mW and $8^2/4$ = 16mW.

    [Method 2]: The total resistance is is 7 kΩ so the current flowing in the circuit is 14/7 = 2mA. The voltage across a resistor is I x R which, in for these resistors, gives 2 x 1 = 2V, 2 x 2 = 4V and 2 x 4 = 8V. The power dissipated is V x I which gives 2 x 2 = 4mW, 4 x 2 = 8mW and 8 x 2 = 16mW. The current through the voltage source is 2 mA, so the power it is supplying is V I = 14 x 2 = 28mW. This is, inevitably, equal to the sum of the power dissipated by the three resistors: 4 + 8 + 16 = 28.

11. [Method 1]: The resistors are in parallel and so form a current divider: the 21mA will divide in proportion to the conductances: 1 mS, 0.5mS and 0.25 mS. The total conductance is 1:75 mS, so the three resistor currents are 21 x 1/1.75 = 12 mA, 21 x 0.5 /1.75 = 6mA and 21 x 0.25 /1.75 = 3 mA. The power dissipated in a resistor is $I^2 R$ which gives 12^2 x 1 = 144mW, 6^2 x 2 = 72mW and 3^2 x 4 = 36mW.

    [Method 2]: The equivalent resistance of the three resistors is 1 / (1/1+1/2+1/4) = 4/7 kΩ. Therefore the voltage across all components in the parallel circuit is 21 x 4/7 = 12V. The current through a resistor is V/ R which gives 12/1 = 12 mA,12/2 = 6mA and 12/4 = 3 mA. The power dissipated in a resistor is V x I which gives 12 x 12 = 144mW, 12 x 6 = 72mW and 12 x 3 = 36mW. The power supplied by the current source is 12 x 21 = 252mW which as expected equals 144 + 72 + 36.

12. The resistors form a potential divider, so Y/X = 4/(R_1+4). So we want 4/(R_1+4) = 1/4 => R1 + 4 = 16 => R1 = 12 kΩ.

13. The resistors form a potential divider, so Y/X = R2/(R1+R2) = 1/10.

    So we want R2/(R1+R2) = 1/10 and R1 + R2 = 10 MΩ. Substituting one into the other and cross-multiplying gives 10 x R2 = 10MΩ => R2 = 1 MΩ. Substituting this into the simpler of the two initial equations gives R1 = 10 - 1 = 9 MΩ.

14. (a) 3 k = 1.5 k + 1.5 k = 3.3 k||33 k, (b) 4 k = 3.9 k + 100, (c) 3.488 k = 3:9 k||33 k.

    To make an exhaustive search for creating a resistance of R, you need to consider two possibilities:

    (i)   for two resistors in series, the largest of the two resistors must be in the range [1/2R,R] or
    (ii)   for two resistors in parallel, the smallest resistor must be in the range [R, 2R].

    In both cases there are at most four possibilities, so you need to consider up to eight possibilities in all. So, for example, for R = 3:5 k, we would consider the following possibilities: (i) 1.5 k+1.8 k = 3.3 k, 1.8 k+1.8 k = 3.6 k, 2.2 k + 1.2 k = 3.4 k, 2.7 k + 0.82 k = 3.52 k and (ii) 3.9 k||33 k = 3.488 k, 4.7 k||15 k = 3.579 k, 5.6 k||10 k = 3.59 k, 6.8 k||6.8 k = 3.4 k. The choice with least error is the one given above.

    Since we are interested in percentage errors, we need to consider the ratio between resistor values. The largest ratio between successive resistors is the series is 15 /12 = 1.25 (this includes the wraparound ratio of 100/82 = 1.22). The worst-case percentage error will arise if our target resistance is the mean of these two values, 13.5. The percentage error in choosing either one is then 1.5/13.5 = 11:1%.